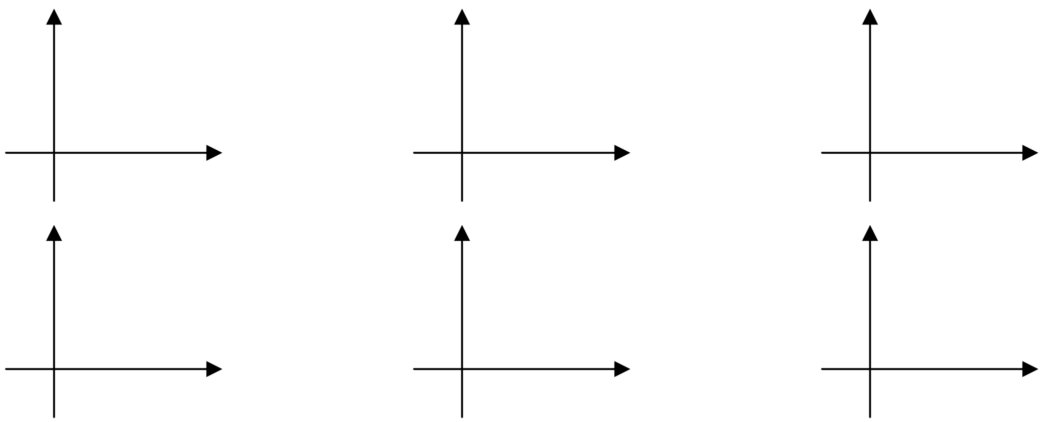
Math 214 – Introductory Statistics **Summer 2008**
6-19-08 Class Notes **Sections 6.2, 6.3**
6.3: 7-45 odd

<u>The Standard Normal Distribution</u>

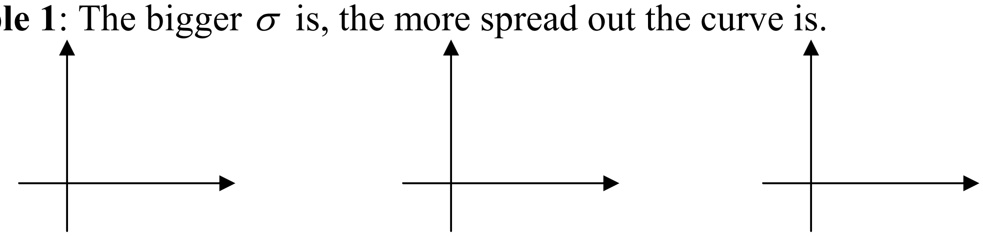
When we graph a distribution (with say the histogram), we can get a wide variety of shapes. They can be *right-skewed* (majority of the data values to the left), *left-skewed* (majority of the data to the right), *bimodal*, *u-shaped*, or even *uniform*. However, many random variables have distributions that are similar in shape. The most common shape for a distribution is *bell-shaped*.



Since so many real-world distributions are bell-shaped, this distribution is called the ***normal distribution***. It's also called the ***Gaussian distribution*** after the German mathematician Carl Gauss (pronounced "gowse") who first discovered its formula. For the following sections, we'll focus on continuous random variables.

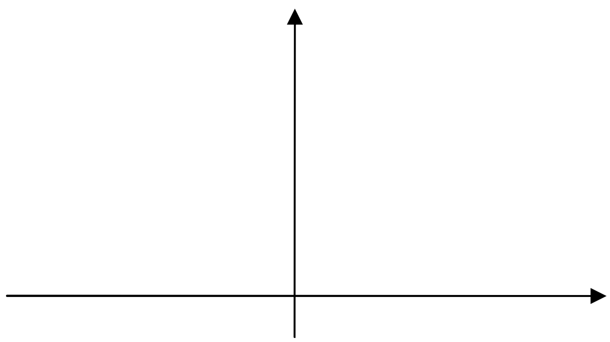
Here are several properties of the normal distribution:
(1) Bell-shaped, continuous, and symmetric about the mean
(2) The mean, median, and mode are all equal
(3) The curve never touches the x-axis
(4) The total area under the curve is 1
(5) Approx. 68% of the area lies within one s.d. of the mean, 95% lies within two s.d., and 99.7% lies within three s.d.

Example 1: The bigger σ is, the more spread out the curve is.



Notice that there are infinitely many normal distributions, one for each mean and standard deviation. However, there is a way to convert one into another, so we only need to understand one normal distribution, the ***standard normal distribution*** with $\mu = 0$ and $\sigma = 1$. We usually use the letter z to denote the variable of a standard normal distribution.

Now, for a given continuous random variable, there are infinitely many possible values it can take on. So the probability that z equals any particular value is 0. However, we can determine the probability that z lies in a certain interval. For example, since the standard normal distribution is symmetric with mean 0, the probability that a particular z-value is great than 0 is .50. In general, the probability that z lies in some interval, say $a < z < b$, is the area under the standard normal curve between a and b.



To find these areas, we use Table E (at the front of the textbook).

Example 2: Find the probability that z lies in the interval $[0,1.58]$. This is usually denoted as $P(0 < z < 1.58)$.

$$P(0 < z < 1.58) = .4429$$

Example 3: Find $P(z < 2.07)$.

$$P(z < 2.07) = .5 + .4808 = .9808$$

Example 4: Find $P(-.55 < z < 0)$.

$$P(-.55 < z < 0) = .2088$$

Example 5: Find $P(-1.11 < z < .75)$.

$$P(-1.11 < z < .75) = .3665 + .2734 = .6399$$

Example 6: Find $P(z > 2.40)$.

$$P(z > 2.40) = .5 - .4918 = .0082$$

Example 7: Find $P(z < -1.08)$.

$$P(z < -1.08) = .5 - .3599 = .1401$$