Math 214 – Introductory Statistics **Summer 2008**

6-27-08 Class Notes

Sections 8.3, 8.4
8.3: 1, 5, 9, 11
8.4: 5, 7, 13, 15

<u>The *z*-test and the *t*-test</u>

If our sample size is greater than or equal to 30, we use the z-test (as in the past). If our sample size is small (less than 30) we use the t-test.

Recall:

<u>One-Tail –vs– Two-Tail</u>

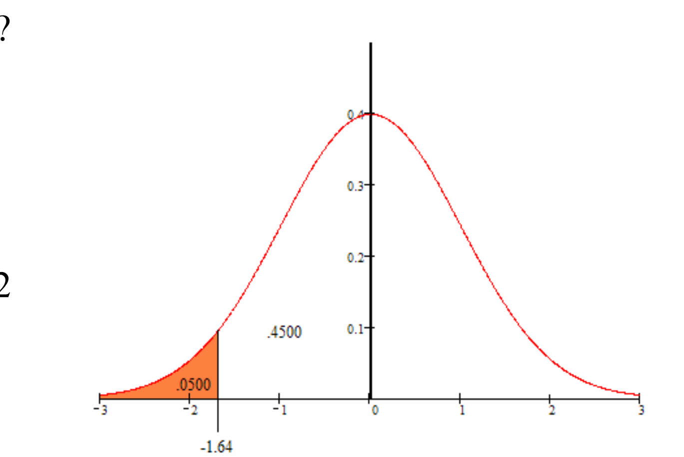
(a)	(b)	(c)
$H_0 : \mu \geq c$	$H_0 : \mu \leq c$	$H_0 : \mu = c$
$H_1 : \mu < c$	$H_1 : \mu > c$	$H_1 : \mu \neq c$
Left-tailed	**Right-tailed**	**Two-tailed**

Example 1: It has been reported that the average credit card debt for college seniors is $3262. The student senate at a large university feels that their seniors have much less debt, so it conducts a study of 50 randomly selected seniors and finds that their average debt is $2995 with a standard deviation of $1100. With $\alpha = 0.05$, is the student senate correct?

$H_0 : \mu \geq 3262$

$H_1 : \mu < 3262$ (claim)

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{2995 - 3262}{1100 / \sqrt{50}} = -1.72$$



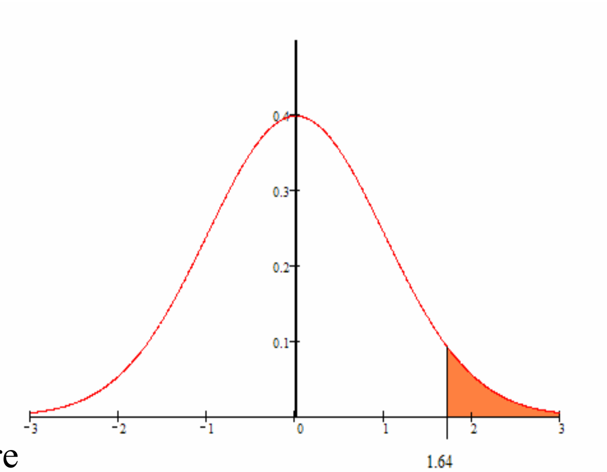
Since the z-value lies in the critical region (the reject region) we are able to reject the null hypothesis. The claim made by the student senate was the alternate hypothesis, so we can say "There is enough evidence to support their claim."

Notice that the claim in the problem can be the null hypothesis *or* the alternate hypothesis. But in either case, we test the null. We then have to be able to summarize our finding into a conclusion.

Example 2: The average production of peanuts in the state of Virginia is 3000 pounds per acre. A new plant food has been developed and is tested on 60 individual plots of land. The mean yield with the new food is 3120 pounds per acre with a standard deviation of 578 pounds. At $\alpha = 0.05$, can we conclude that the average production has increased?

$H_0 : \mu \le 3000$
$H_1 : \mu > 3000$ (claim)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3120 - 3000}{578/\sqrt{60}} = 1.61$$

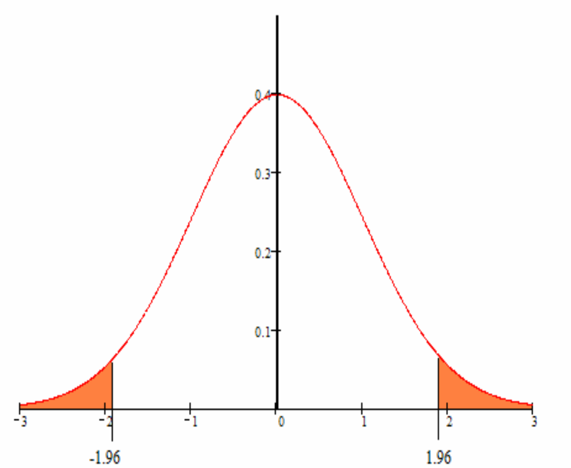


Since the z-value does not lie in the critical region (the reject region), we cannot reject the null hypothesis. So there is *not* enough evidence to support the claim that peanut production has increased.

Example 3: A real estate agent claims that the average price of a home sold in Beaver County, Pennsylvania is $60,000. A random sample of 36 homes sold in the county is selected and the mean price is $82,495 with a standard deviation of $76,024. At $\alpha = 0.05$, is there enough evidence to reject his claim?

$H_0 : \mu = 60,000$ (claim)
$H_1 : \mu \ne 60,000$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{82495 - 60000}{76024/\sqrt{36}} = 1.78$$

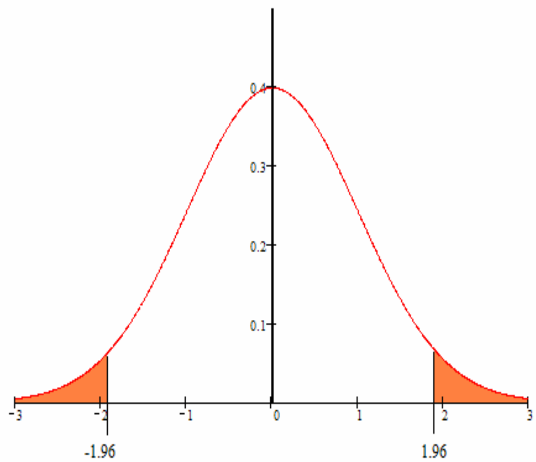


Since the z-value does not lie in the critical region (the reject region), we cannot reject the null hypothesis. So there is *not* enough evidence to reject his claim.

Example 4: A researcher claims that the yearly consumption of soft drinks per person is 52 gallons. In a sample of 50 randomly selected people, the mean was 56.3 gallons and the standard deviation was 3.5 gallons. With $\alpha = 0.05$, is the researchers claim valid?

$H_0 : \mu = 52$ (claim)
$H_1 : \mu \neq 52$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{56.3 - 52}{3.5/\sqrt{50}} = 8.69$$



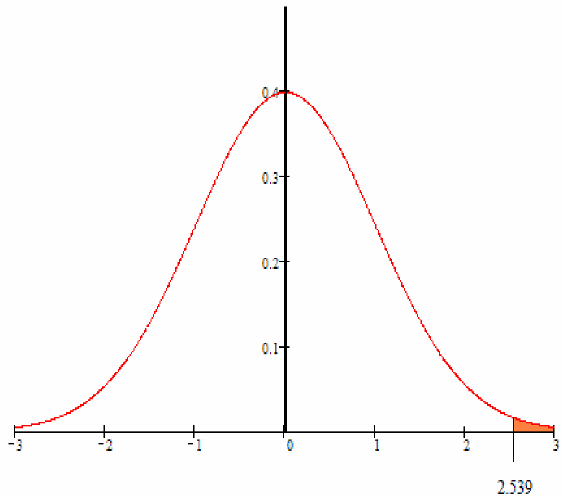
Nope! The z-value is WAY into the reject region. So we reject the null hypothesis and therefore his claim.

<u>Summary of Decisions</u>

(1) If claim is H_0 and you can reject H_0,
"There is enough evidence to reject the claim."
(2) If claim is H_0 and you cannot reject H_0,
"There is not enough evidence to reject the claim."
(3) If claim is H_1 and you can reject H_0,
"There is enough evidence to support the claim."
(4) If claim is H_1 and you cannot reject H_0,
"There is not enough evidence to support the claim."

Now, if the sample size is small, we have to use a similar test, but because the sample size is small, we have to determine the critical values by using a chart (Table F).

Example 5: The average undergraduate cost for tuition, fees, and room and board for two-year institutions last year was \$13,252. The following year, a random sample of 20 institutions had a mean of \$15,560 with a standard deviation of \$3500. At $\alpha = 0.01$, is there sufficient evidence to conclude that the cost has increased?



$H_0 : \mu \leq 13{,}252$
$H_1 : \mu > 13{,}252$ (claim)

$$t = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{15560 - 13252}{3500/\sqrt{20}} = 2.95$$

From Table F, we see that the critical value is 2.539. Since our t-value lies in the critical region, we can reject the null hypothesis. Therefore, there is enough evidence to conclude that costs have increased.