

MATH 509
LOGIC AND FOUNDATIONS OF MATHEMATICS
FOR TEACHERS

Nicholls State University, Fall 2004

Instructor: Dr. Brian Heck. My office is 106-E Peltier Hall and my office phone number is 448-4383. Office hours are: 8:30 - 9:30 a.m. and 9:00 a.m. - 1:00 p.m. T, TH. Please come by if you have any questions. Also, my email is brian.heck@nicholls.edu.

Prerequisites: Math 265 and 358.

Course Description (catalog): Cornerstone course normally taken in first semester of graduate study. Developing and evaluating arguments and proofs, the use of various types of reasoning, methods of proof, making and investigating conjectures.

Text: *On Formally Undecidable Propositions of Principia Mathematica and Related Systems* by Kurt Gödel (Dover 1992 ISBN 0-486-66980-7).

Course Objectives: At the completion of the semester, a student will be able to:

- explain the historical developments of logic and set theory
- form sound arguments using the notions of set theory, functions, equivalence relations, and logic
- prove theorems in many fields of mathematics using the various methods of proof including direct, indirect, and mathematical induction,
- evaluate an argument and/or proof for its logical validity,
- use deductive reasoning to form conjectures

This course is primarily designed for secondary and post-secondary teachers. The ability to make coherent and logical arguments is necessary for effective teaching at all levels. Understanding students' logical errors and how to avoid them is also essential. Mathematically speaking, theorem conjecturing and proving (or disproving) is the heart of advanced mathematics. This course is intended to supply the prospective teachers with an increased ability to analyze, evaluate, and form logical arguments.

We will begin by covering the basics of logic and theorem proving. This will be followed by a careful analysis of the history of the axiomatic method. From Euclid and Archimedes to Peano and Hilbert, we look at successes and failures in attempts to axiomatize various fields of mathematics. This takes us to the dawn of the 20th century. At the beginning of the 1900's, mathematics underwent a schism. Led primarily by Hilbert, Russell, and Brouwer, the foundation of mathematics was being exhaustively

studied, debated and fought over. This remained largely unresolved until 1933 when a 25-year old Austrian by the name of Kurt Gödel published his epoch-making paper *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*. The punch line of this paper was that in any field of mathematics and under any set of axioms there were statements that were undecidable, in other words neither provable nor unprovable. Hence the axiomatic method was doomed to failure. We will examine this amazing paper. Finally, as time allows, we will apply the principles we have learned to several recent fields of mathematics selected from computing, graph theory, coding theory, and information theory. I hope you will enjoy this semester as much as I know I will.

Course Evaluation: You will have four (4) equally weighted components to your grade: boardwork, problem sets, a midterm exam, and a final exam. The two exams will each be closed notes/text and quasi-comprehensive.

Attendance/Expectations: Students are expected to attend all classes. Excuses for missed classes will be handled on an individual basis. In any event, each student is responsible for all material covered in class. It is assumed that you are attending this university because you have a desire for higher learning. It is therefore expected that you will pay attention, be respectful of your instructor and fellow students, and follow the Code of Student Conduct. Instances of academic dishonesty will be dealt with severely. If you are caught cheating, you will fail this course. Similarly, if you are a disruptive presence in the classroom, you will be dropped from the class.

Students with disabilities: If you have a documented disability that requires assistance, you will need to register with the Office of Disability Services for coordination of your academic accommodations. The Office of Disability Services is located in Peltier Hall, Room 100-A. The phone number is (985) 448-4430 (TDD 449-7002).