

**Math 510**  
**Number-Theoretic and Discrete Structures**  
**Fall 2009 Notes**

Dr. Brian K. Heck  
Nicholls State University

## INTRODUCTION

This set of notes is designed for a graduate level number theory course at Nicholls State University entitled “Number-Theoretic and Discrete Structures”. They are suitable for a one semester course. The student should have familiarity with logic, set theory, and have theorem-proving experience. A course in abstract algebra, while not essential, would also be helpful.

We begin by studying the building blocks of the real number system, the prime numbers. We will also study the integers and their relationship with the rationals, congruences, and Carl Gauss’ quadratic reciprocity. These topics then lead us into quadratic number fields.

Finally, as time allows, we will cover various discrete structures such as graphs, lattices, and/or recursive relations. In particular, we may find the time to relate certain graphs (circulant graphs) to the study of algebraic number theory. I hope you will enjoy this semester as much as I know I will.

# TABLE OF CONTENTS

## Introduction

### I Prime Numbers

Section 1: The Building Blocks of Numbers

Section 2: Divisibility and Unique Factorization

Section 3: Types of Primes

### II Congruences

Section 1: Linear Congruences

Section 2: Quadratic Reciprocity

Section 3: Cryptography

### III Algebraic Equations

Section 1: Diophantine Equations

Section 2: Algebraic Numbers

Section 3: Gaussian Integers

Section 4: Algebraic Number Theory

### IV Recursive Relations

### V Graph Theory

Section 1: Introduction to Graphs

Section 2: Circulant Graphs

Section 3: Redei-Reichardt Theorem