INTRODUCTION - What is The Calculus?

There are many different calculi: calculus of variations, differential calculus, and vector calculus, among others. In fact the word "calculus" is not just a math term but an English word as well:

> Random House: cal' cu lus *n*. A method of calculation by a special system of algebraic notations.

So what is the calculus and why does it stand out in the history of mathematics? Periodically throughout the history of mathematics, certain problems arose that appeared exceedingly difficult, paradoxical, even unsolvable. They would inspire many to attempt to find their solutions, only to resist all methods of their age. Eventually, as mathematics grew more sophisticated, and years of effort slowly began to pay off, solutions were found. Squaring the circle, doubling the cube, trisecting an angle, Fermat's Last Theorem, and the Four Color Problem are just a few such examples. In each of these cases, the resolution to the problem eventually came, usually within a few hundred years. But there were two problems that originated in ancient Greece roughly 2,500 years ago that remained confusing and unsolved for two millennia.

(1) How do you find the equation of the line tangent to a curve at a given point?
(2) How to you find the area of a region bounded by a given curve?

Both of these questions are fairly easy to state, easy to understand, and specific. Yet when they first occurred to mathematicians, the mathematics of the day was insufficient to answer them. That does not deter mathematicians however; unsolvable problems are like the siren's call. If these problems had been mere curiosities that had few applications if any, mathematicians would have still pursued their solutions for years, but indeed quite the opposite proved true. Time after time, problems were discovered that essentially referred back to one of these two. How do you compute the area of a circle? How do you find the rate of growth of a changing substance? How do you approximate pi? How do you find the curvature of a curve? Where does a function achieve its maximum and minimum? Many other basic questions in geometry, algebra, theory of equations, physics, and astronomy also boiled down to one of the ancient Greek problems. By the 16^{th} and 17^{th} centuries many individual cases had been resolved, a few particular tangent lines could be found for instance, but the general theory was nonexistent. In stepped Isaac Newton and Gottfried Leibniz, albeit separately. In the 1670's general solutions to both problems were discovered and in fact the problems themselves were united into one theory; The Calculus.

Calculus as we know it is essentially broken into two halves: differential calculus and integral calculus. We will see that while modern day calculus classes begin with differentiation and proceed to integration, interestingly enough integration historically developed first. The fundamental notions of both branches – derivatives and integrals – involve the paradoxical idea of infinity and infinitesimals, and this is what proved to be the difficulty for so long.

We will begin in Chapter I with Newton and Leibniz. Both were accomplished men even putting aside their amazing contributions to the development of The Calculus. Their many achievements could fill volumes, but we will just highlight their greatness. But Newton himself once said that if he had seen farther than other men, it was only because he had stood on the shoulders of giants. So we will spend the three chapters thoroughly covering the progress made by others from early Greeks (like Eudoxus and Archimedes) to contemporaries of Newton and Leibniz (Wallis and Barrow). Finally, as time allows we will address how calculus evolved in the days after Newton/Leibniz.

With each set of notes, I will assign a reading that covers the relevant historical and biographical information and supplement with the mathematics (and perhaps a few historical nuggets of my own). So let's get to it….