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Scientists speculate that man has existed for hundreds of thousands of years. But it was not until fairly late in our development as a species that we created civilization. Tools, the alphabet, money, and farming are all thought to be discovered/developed around the same time, roughly 4000 BC to 5000 BC. However, in 1937 a wolf’s bone was found in the former Czechoslovakia that had evidence of counting from 30,000 BC. On this bone were 55 notches, arranged by fives, in two groups (one containing 35 notches, the other 20). This suggests that man has had the ability to count long before he could write, farm, or build. In fact anthropologist George Murdock has noted that numerals are one of 72 items existing in every human culture known to man. It has been argued that counting even predates mankind, as many animal species appear to have primitive counting abilities. Magpies can recognize the size of a group of 5 or 6 objects. The chimp knows that 5 objects are less than 6. Certain insects show a similar power to recognize the relative sizes of small groups. Experiments with crows have shown that they can distinguish between sets with up to 4 elements.

A word should also be said regarding our familiar base 10. The fact that most cultures utilized this base is most certainly a result of our having 10 fingers on our hands. But many cultures settled on different bases. There is evidence of cultures that used base 2, base 3, base 4, base 5, base 6, base 12, base 20, and base 60. A summary of some of these cultures and the evidence is described below:

<table>
<thead>
<tr>
<th>Base</th>
<th>Culture/Country</th>
<th>Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Australia</td>
<td>The words for the natural numbers translate as “1, 2, 2 and 1, two 2’s, much”.</td>
</tr>
<tr>
<td>3</td>
<td>Phoenician</td>
<td>Early notations grouped are by 3’s.</td>
</tr>
<tr>
<td>4</td>
<td>South American tribes</td>
<td>The words for the natural numbers translate as “1, 2, 3, 4, 4 and 1, 4 and 2, etc”.</td>
</tr>
<tr>
<td>5</td>
<td>South American tribes</td>
<td>The words for the natural numbers translate as “1, 2, 3, hand, hand and 1, etc”.</td>
</tr>
<tr>
<td></td>
<td>Paraguay</td>
<td>The word for 5 translates to “fingers on one hand” and for 10 “both”.</td>
</tr>
<tr>
<td>3, 4, and 5</td>
<td>Yukaghirs of Siberia</td>
<td>The words for the natural numbers translate as “1, 2, 3, 3 and 1, 5, two 3’s, 1 more, two 4’s, 10 with 1 missing, 10”.</td>
</tr>
<tr>
<td>6</td>
<td>South Bretagne</td>
<td>The word for 18 translates to “three sixes”.</td>
</tr>
<tr>
<td>12</td>
<td>English</td>
<td>Many examples such as units inches,</td>
</tr>
</tbody>
</table>
ounces, pence, hours, months, dozen, and gross. Also, the natural base 10 names for the numbers following 10 logically would be something like 1-teen and 2-teen, but instead we have eleven and twelve.

20 French The word for 80 translates to “four twenties”.

Orinoco The word for 20 translates to “one man”, 21 translates to “one to the hands of the next man”

English The unit score.

60 Babylon We’ll see extensive evidence of this when we study this culture in Section 1 of Chapter 1.

English The units minutes, seconds, and degrees.

However, mathematics is much more than number systems and counting. Mathematics as an intellectual endeavor developed roughly along with language and writing. Elementary mathematics, specifically arithmetic and geometry, first developed in ancient societies as an empirical science. People needed to be able to measure their land, engineer and construct buildings, and have commerce and trade. The ability to find a method that seemed to accomplish what was needed was all that was important. If the procedure worked, the people accepted it. There is no evidence that anyone sought to prove anything. Eventually, the desire to know why things worked, or how they worked became more and more pronounced. As mathematics developed, it grew into not just an empirical science, but also a branch of philosophy and logic.

We will begin our studies, just as mathematics did, in the ancient civilizations of Egypt and Babylon. While all of the earliest civilizations did mathematics, it was the Egyptians and the Babylonians who had the greatest success recording and preserving their accomplishments. We will also learn about the other major early civilizations that thrived primarily in China and India. Then we will cover the contributions of the Arabians, the Europeans, and finally the Americans.

Finally, I want to caution you that any “history of mathematics” course is necessarily going to leave out many interesting cultures, topics, theories, people and stories. As you will see, mathematics has a long and rich history ripe with fascinating people, controversies, and anecdotes. It would take more than 8 weeks to adequately cover the 6,000 years of history (in fact, even 80 weeks might not be enough). I will focus on the topics and people that are most important (and most interesting) in my opinion. I hope you will enjoy this course as much as I know I will.